New Approach of Solving Time-Domain Free Space Wave Propagation

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Abstract— **In this paper, a numerical simulation by a new high speed low order FDTD (HSLO-FDTD) method will be conducted to simulate one dimensional free space wave propagation of 2.4 GHz Gaussian pulse. The efficiency of the new schemes are analyze and compared with the standard FDTD method in terms of processing time, phase velocity and global error. The amplitude in volts by both methods is also displayed. Results obtained using the new schemes compare well with published results and solve the problem faster than the standard FDTD method.**

Keywords-FDTD; HSLO-FDTD; numerical simulation of wave propagation

I. INTRODUCTION

In the advancement of computer technology, numerical simulation plays a major role in most of fields in science and technology. The method accelerates and facilitates research and industrial development in those fields. The demands of advanced wireless communication devices in recent high technology world has increase the need of tools that facilitate research and development in the field of electromagnetic. The finite difference time domain (FDTD) method is one of the most popular tool in simulating electromagnetic problem, because it covers many applications[1], such as antennas, wireless and wired communication, high speed electronic circuit, biomedical, semiconductors and etc. The wave propagated in free space from a transmitter to a receiver is the ultimate event of wireless communication. All of those problems are solved via Maxwell equations ([1],[2]). Beside FDTD, Transmission Line Method (TLM)[3] is another method that can be implemented in the time-domain.

II. SOME BRIEF LITERATURE ON INCREASING THE SPEED OF FDTD METHOD

In computational electromagnetic (CEM), FDTD refers to finite difference approximation to the Faraday's and Ampere's laws using second order accurate in time and space. The method was first proposed in [4]. Yee used an electric field (E) which was offset both spatially and temporally from a magnetic field grid to obtain update equations that yield the present fields throughout the computational domain in term of past fields. The method was further developed to solve electromagnetic scattering from a dielectric cylinder[5]. This method is the most commonly used to solve problem in time-domain because of its simplicity and directly adapted to homogeneous problem.

Since then, the method has become one of the most powerful Maxwell equations solvers of electrodynamics. It has been implemented on various applications ([6]-[18]). The algorithm simplicity, robustness, and potential for high complexity afforded by FDTD have prompted an extraordinary level of interest in this technique. However, there are drawbacks in the method. One of the drawbacks is it needs a long processing time to simulate problems.

To increase the speed of the method, some researchers apply higher-order scheme in FDTD method. Lan, Liu and Lin [19] has developed a second order accuracy in time and fourth order in space. This new scheme is then compared to FDTD by modeling plane-wave pulse propagating through free-space. Result show that the higher-order method reduces the numerical dispersion and has improved stability. Georgakopoulos, Birtcher, Balanis and Renaut [20] apply the FDTD (2, 4) to a wave propagating problem. In the paper, they used the same gridding concept in [21]. The only difference is that they implement FDTD (2, 4) at the coarser grid. Propokidis and Tsiboukis [22] have implement FDTD (2, 4) to simulate a lossy dielectric problem. All of the experimental result shows that FDTD (2, 4) can simulate accurately with coarser grid mesh than the FDTD (2, 2). The implementation of higher-order truncation to Maxwell equations increase the complexity of the method, however by solving the problem in coarser grid will increase the speed of the processing time.

The advancement of multiprocessor technology also influences the development of high speed FDTD algorithms. Perlik, Opsahl and Taflove [23] develop parallel FDTD algorithm to predict scattering of electromagnetic fields on a Connection Machine (CM). [24] implement a parallel FDTD algorithm on network of Transputers to simulate electromagnetic waves of a rectangular antenna. Jensen, Fijany and Rahmat-Samii

[14] proposed a new design of parallelism, in both spatial and time. They solve circular scatterer but the design is merely analyzed theoretically. Nguyen, Zook and Zhang [25] solves electromagnetic scattering problem using domain decomposition FDTD with heterogeneous network of workstation which composes of 4 SUN SPARCstation and four IBM RS/6000. Zhenghui, Benqing and Zejie [26] describe the strategy used for parallel implementation of the FDTD algorithm on cluster of workstation with two heterogeneous PC (Pentium-II 266/64Mb and Pentium-II 200/64Mb) and a workstation by PVM parallel software to solve one dimensional free space problem. Yang, Liao, Jen and Xiong [27] proposed new design of decomposition for implementing parallel FDTD on domain decomposition using MPI library to analyze coupling model of pulse into slot. The researcher decomposes the whole domain into several sub-domains according to features of the problem. Moreover, each sub-domain may have its own lattices independently to suit the special shapes.

In this research, we increase the speed of FDTD processing time via different approach. The new method which is called the High Speed Low Order FDTD (HSLO-FDTD) method will solve a one dimensional wave propagating in free space problem on the same mesh size used by the standard FDTD by a single processor machine.

III. FREE SPACE MAXWELL EQUATIONS

Let's consider the Maxwell equations for free space below.

$$\frac{\partial E}{\partial t} = \frac{1}{\varepsilon_0}\nabla \times H \tag{1}$$

$$\frac{\partial H}{\partial t} = -\frac{1}{\mu_0}\nabla \times E \tag{2}$$

where E,H,ε_0 and μ_0 are the electric fields, magnetic fields, electric permittivity and magnetic permeability, respectively. For the one-dimensional case using E_x and H_y, the (1) and (2) become

$$\frac{\partial E_x}{\partial t} = -\frac{1}{\varepsilon_0}\frac{\partial H_y}{\partial z} \tag{3}$$

$$\frac{\partial H_y}{\partial t} = -\frac{1}{\mu_0}\frac{\partial E_x}{\partial z} \tag{4}$$

These are the equations of a plane wave with the electric field oriented in the x-direction, magnetic field in the y- direction, and traveling in the z-direction. For further details, see [1].

IV. HIGH SPEED LOW ORDER FINITE-DIFFERENCE TIME-DOMAIN METHOD

The HSLO-FDTD method was developed by borrowing the concept implemented in Modified Explicit Group (MEG) introduced recently in [28], which is the extension of the concept of the half-sweep iterative method propose in [29] through the Explicit Decoupled Group (EDG) iterative method. Both of the Modified Explicit Group and the Explicit Decoupled Group are classified as iterative method and were used to solve elliptic type of problem.

In this research, we modify the concept used in MEG method and apply it to develop HSLO-FDTD for solving the free space Maxwell equations in time-domain. The iterative concept in MEG is ignored because there is no matrix in HSLO-FDTD method to be solved. By taking central difference approximations as below,

$$\frac{\delta F(i)}{\delta x} = \frac{F^n(i+\frac{m}{2}) - F^n(i-\frac{m}{2})}{m\Delta x} + O(\Delta x^2) \tag{5}$$

for spatial derivatives and

$$\frac{\delta F(i)}{\delta t} = \frac{F^{n+\frac{1}{2}}(i) - F^{n-\frac{1}{2}}(i)}{\Delta t} + O(\Delta t^2) \tag{6}$$

for temporal derivatives in (3) and (4) yields

$$\frac{E_x^{n+\frac{1}{2}}(k) - E_x^{n-\frac{1}{2}}(k)}{\Delta t} = -\frac{H_y^n(k+\frac{m}{2}) - H_y^n(k-\frac{m}{2})}{m\varepsilon_0\Delta x} \tag{7}$$

$$\frac{H_y^{n+1}(k+\frac{m}{2}) - H_y^n(k+\frac{m}{2})}{\Delta t} = -\frac{E_x^{n+\frac{1}{2}}(k+m) - E_x^{n+\frac{1}{2}}(k)}{m\mu_0\Delta x} \tag{8}$$

By rearranging (7) and (8) above the same way as standard FDTD scheme, yields

$$\widetilde{E}_x^{n+\frac{1}{2}}(k) = \widetilde{E}_x^{n-\frac{1}{2}}(k) - D^*\left(H_y^n(k+\frac{m}{2}) - H_y^n(k-\frac{m}{2})\right) \tag{9}$$

$$H_y^{n+1}(k+\frac{m}{2}) = H_y^n(k+\frac{m}{2}) - D^*\left(\widetilde{E}_x^{n+\frac{1}{2}}(k+m) - \widetilde{E}_x^{n+\frac{1}{2}}(k)\right) \tag{10}$$

where

$$D^* = \frac{\Delta t}{m\sqrt{\varepsilon_0\mu_0}\Delta x}$$

which m is odd number. See that (9) and (10) is a generalized form of FDTD method where when $m = 1$, it is the standard FDTD, when $m = 3$, it is the HSLO(3)-FDTD, when $m = 5$, it is the HSLO(5)-FDTD, and so on. By using (9) and (10), we only calculate $\frac{1}{m}$ of node points in the entire solution domain from 0 to T time steps.

Equations (9) and (10) with $m = 3$, will be used to solve problem in solution domain given by Fig.1(b) with the black square and circle are the magnetic and electric fields respectively have to be solved in the main HSLO(3)-FDTD algorithm. The uncalculated node, the white square and circle will be solved later only at T after the entire black node have been calculated. The standard FDTD will be executed on solution domain given by Fig. 1(a).

V. NUMERICAL EXPERIMENT AND RESULTS

The effectiveness of HSLO (3)-FDTD method is analyzed by executing a one dimensional free space wave propagation problem with Gaussian pulse as the point source. We will generate a 2.4 GHz Gaussian pulse at the middle of the solution domain of 2 meter, truncated with simple absorbing boundary condition. To ensure the accuracy of the simulated result, the solution domain is discretize into 600 grid points

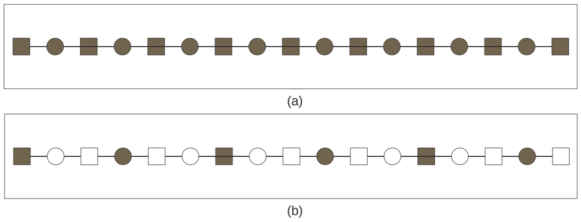

Fig. 1. (a) Solution domain for standard FDTD method and (b) Solution domain for HSLO (3)-FDTD method

with cell size of 0.0033 meter and time slice size of 5.5×10^{-12} ns. The wavelength of frequency 2.4 GHz is divided by 37 cells. This means that we use $\Delta t = \frac{\Delta x}{2\dot{c}_0}$ where c_0 is wave velocity in free space. The experiment was run on Intel Pentium 3 of Mobile CPU 1 GHz 727 MHz 256 MB of RAM with LINUX operating system.

The result of simulation is given in Fig. 2 and 3. From those figures, HSLO (3)-FDTD results are very well suited with standard FDTD. These figures (Fig. 2 & 3) shows the behavior of wave propagation in free space from the point source until absorbed by the boundary at the truncated lattices. In this experiment, we use a simple absorbing boundary condition that is always used in one-dimensional problem to truncate the lattice of solution domain.

The comparison of execution time is given in Fig. 4 and we can see that the new scheme simulate the problem faster than the standard FDTD scheme with 57.14% to 64.64% reduction in processing time. Even though in this experiment we only manage to save 10 ms, but if we are solving longer solution domain or simulating for higher time level, we will be able to save more time. The HSLO (3)-FDTD method also give a more stable approximation of amplitude than standard FDTD (refer Fig. 5). However, there exists a small reduction in accuracy of approximation by HSLO (3)-FDTD method. The reductions in accuracy are shown in Fig. 6, but this reduction is very small and not significant. Fig. 6 shows differences in power density (W/m^2) between standard FDTD and HSLO (3)-FDTD. The wave simulated by HSLO (3)-FDTD method travel faster (0.67%) than the exact wave phase velocity in free space. Fig. 3 shows both simulated wave are absorbed well at both boundaries. The accuracy of absorption is calculated by simulating problem in wider solution domain by both method and compares the result with the previous simulation. This method of comparison is also used by [20]. The accuracy of absorption at boundaries is displayed in Table I. This table shows that HSLO-FDTD has bigger global error than the standard FDTD, but the errors are still very small and not significant.

VI. CONCLUSION

The HSLO-FDTD method gives us the opportunity to solve $\frac{1}{m}$ grid point of the solution domain in the main loop of HSLO-FDTD and the remaining point only at the required time step. This approach has increase the speed of FDTD

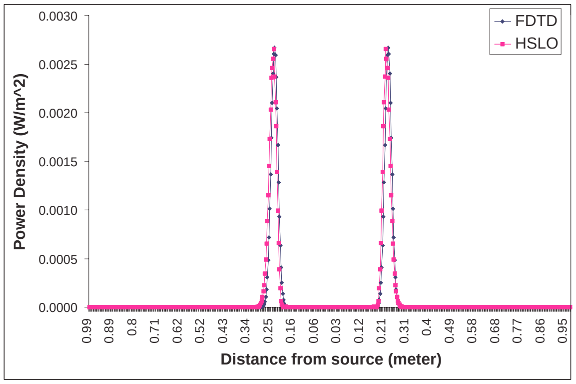

Fig. 2. Wave propagation from the centre of solution domain at 1ns

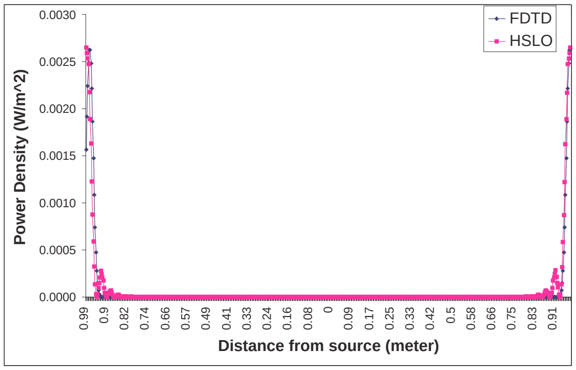

Fig. 3. Wave propagation from the centre of solution domain at 3.47 ns

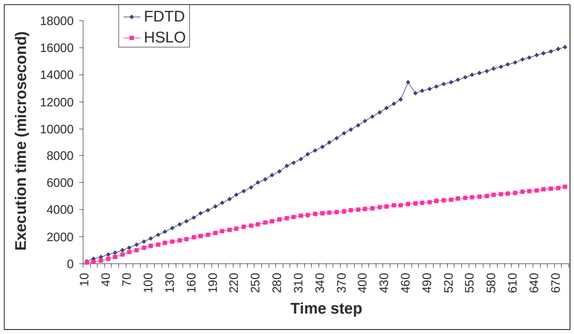

Fig. 4. Comparison of HSLO(3)-FDTD and standard FDTD processing time

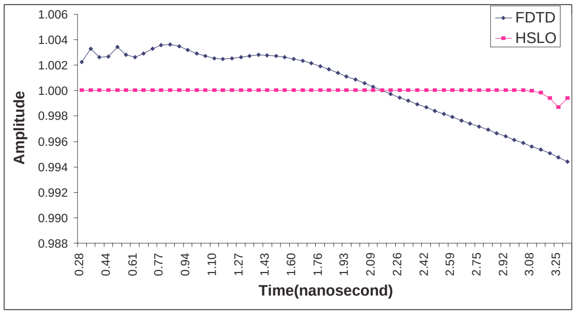

Fig. 5. Amplitude in volts per meter for standard FDTD and HSLO(3)-FDTD

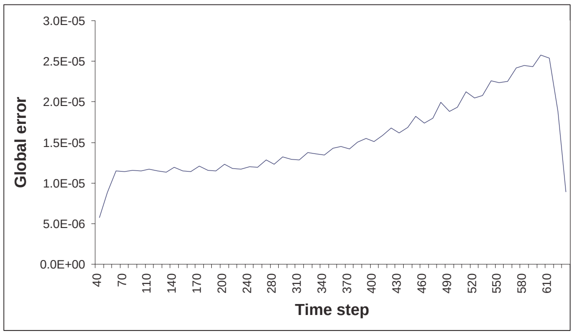

Fig. 6. Global error between HSLO (3)-FDTD and standard FDTD methods

TABLE I

GLOBAL ERROR FOR STANDARD FDTD AND HSLO(3)-FDTD CAUSED BY NON-PHYSICAL REFLECTION AT BOUNDARIES

Method	**Simulation time(ns)**				
	4.400	4.675	4.950	5.500	Average
Std-FDTD	$1.8e^{-18}$	$1.7e^{-18}$	$1.6e^{-18}$	$1.7e^{-19}$	$1.5e^{-18}$
HSLO(3)-FDTD	$1.0e^{-10}$	$1.6e^{-11}$	$1.2e^{-11}$	$1.1e^{-11}$	$1.1e^{-11}$

algorithm. The Performance of this scheme was tested for problem in one dimensional free space propagation truncated with simple absorbing boundary condition. The major advantages of this scheme is that it requires less processing time and less complexity algorithm than the existing FDTD scheme but there exist a small reduction in its accuracy. It is clearly shown that HSLO-FDTD is a better alternative than FDTD in one-dimension for free space wave propagating simulation.

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